Financial Maths

Section 5.4

Loans and Mortgages

Section 5.4 Loans – Mortgages

If we wish to calculate the repayments needed for a car loan or a mortgage on a house, we use the same procedure for finding the present value as was used in the previous section.

The sum of the present values of each repayment over the given period of time must be equal to the value of the car loan or mortgage.

$$€\text{ Mortgage} = \frac{€\text{ Payment}}{1+i} + \frac{€\text{ Payment}}{(1+i)^2} + \frac{€\text{ Payment}}{(1+i)^3} + \ldots \frac{€\text{ Payment}}{(1+i)^n}$$

$$= \overbrace{\left(\frac{€\text{ Payment}}{1+i}\right)}^{a}\left[\frac{1-\left(\overbrace{\frac{1}{1+i}}^{r}\right)^n}{1-\left(\frac{1}{1+i}\right)}\right]$$

$$\therefore\quad €\text{ Payment} = \frac{€\text{ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$$

(*formulae and tables,* p.31)

i = the effective monthly rate of interest (expressed as a decimal)
n = the number of payments (years/months)
€M = the amount of the mortgage or loan
€P = the repayment per month

Example 1

Calculate the size of the monthly repayments needed for a car loan of €10 000 if the loan is to be repaid over a 5-year term at an effective monthly rate of 0.72%.

$\therefore \quad € \text{ Payment} = \dfrac{€ \text{ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(*formulae and tables*, p.31)

5 years = 60 months ⇒ $n = 60$

This is similar a mortgage Repayment question

$$\text{Payments} = \frac{10000\,(\cdot 0072)(1\cdot 0072)^{60}}{1\cdot 0072^{60} - 1}$$

$$= €205\cdot 84$$

Example 2

Find the monthly repayments required for a mortgage of €150 000, based on an annual rate of 4.5% over 20 years.

$i = AER \qquad R = MER$

$R = \sqrt[12]{1+i} - 1$

$\therefore \quad € \text{ Payment} = \dfrac{€ \text{ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(*formulae and tables*, p.31)

$$R = \sqrt[12]{1\cdot 045} - 1 = 0\cdot 0037$$

20 years = 240 months ⇒ $n = 240$

$$\text{Payments} = \frac{150\,000\,(0\cdot 0037)(1\cdot 0037)^{240}}{(1\cdot 0037)^{240} - 1}$$

$$= €941\cdot 22$$

$\therefore \quad € \text{ Payment} = \dfrac{€ \text{ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(*formulae and tables*, p.31)

Exercise 5.4

1. Calculate the monthly repayments required for a mortgage of €200 000, paid over a 30-year period at an annual interest rate of 6%.

$i = AER \quad R = MER$

$R = \sqrt[12]{1+i} - 1$

$\therefore \quad \text{€ Payment} = \frac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(formulae and tables, p.31)

$MER = \quad R = \left(\sqrt[12]{1.06}\right) - 1 \quad = {}^{*}0.0049$

$30 \text{ years} = 360 \text{ months} \Rightarrow n = 360$

$$\text{Payments} = \frac{200{,}000\,(0.0049)(1.0049)^{360}}{1.0049^{360} - 1}$$

$$\approx €1179.12$$

* The MER correct to 4 decimal places is accurate enough.

2. Alice wants to take out a 20-year mortgage.
The average interest rate over the lifetime of the mortgage is 8% per annum.
Alice can afford repayments of €850 per month.
What is the largest mortgage she can afford?
Give your answer to the nearest €100.

$i = AER \quad R = MER$

$R = \sqrt[12]{1+i} - 1$

$\therefore \quad \text{€ Payment} = \frac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(formulae and tables, p.31)

$20 \text{ years} = 240 \text{ months} \Rightarrow n = 240$

$AER = 8\% \Rightarrow MER = \sqrt[12]{1.08} - 1 = 0.64\%$

let M = mortgage

$$850 = \frac{M\,(0.0064)(1.0064)^{240}}{(1.0064)^{240} - 1}$$

$$850 = (0.0082)\,M$$

$$M = \frac{850}{0.0082} = €104{,}100$$

3. What is the monthly payment, correct to the nearest euro, on a mortgage of €75 000, assuming an interest rate of 8%, for
(a) 20 years (b) 25 years (c) 30 years?
How much interest is paid under each option?

MER ? $R = \sqrt[12]{1+i} - 1$

$R = \sqrt[12]{1.08} - 1 = 0.64\%$

$\therefore$ € Payment $= \dfrac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(formulae and tables, p.31)

$$\text{Payment} = \frac{75000\,(0.0064)(1.0064)^t}{(1.0064)^t - 1}$$

(a) 20 years = 240 months $\Rightarrow t = 240$

$$\text{Payment} = \frac{75000\,(0.0064)(1.0064)^{240}}{(1.0064)^{240} - 1} = €612.48$$

(b) 25 years = 300 months $\Rightarrow t = 300$

$$\text{Payment} = \frac{75000\,(0.0064)(1.0064)^{300}}{(1.0064)^{300} - 1} = €\ 563.50$$

30 years = 360 months $\Rightarrow t = 360$

$$\text{Payment} = \frac{75000\,(0.0064)(1.0064)^{360}}{(1.0064)^{360} - 1} = €\ 533.69$$

4. Your local car dealer offers you two different payment plans to buy a €15 000 car.
Plan A: A 10% discount on the price of the car and a loan on the balance at an annual rate of 9% for 5 years.
Plan B: No discount but a loan for the total price €15 000 at an annual rate of 3% for 5 years.
Which plan should you opt for?

Plan A

Price = €15 000
less 10% discount = 15000 (.9) = € 13500

$\therefore$ € Payment $= \dfrac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(formulae and tables, p.31)

$$\text{Payments} = \frac{13500\,(0.09)(1.09)^5}{(1.09)^5 - 1} = €\ 3470.75$$

Plan B

$$\text{Payments} = \frac{15000\,(0.03)(1.03)^5}{(1.03)^5 - 1} = €\ 3275.32$$

The repayments on Plan B are lower.

5. A woman has saved €250 000 to fund a pension and she now plans to retire. She wishes to draw down equal annual instalments from these savings for the next 25 years.
Assuming a 5% interest rate, calculate the value of each yearly instalment.

This is similar to mortgage repayments

$\therefore$ € Payment $= \dfrac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(*formulae and tables*, p.31)

Present Value of fund = € 250 000
$t = 25$ years
$i = 5\%$
The Value of each instalment ?

Each 'payment' $= \dfrac{250\,000\,(0.05)(1.05)^{25}}{(1.05)^{25} - 1}$

$= €17{,}738.11$

6. Two people want to buy your house. The first person offers you €200 000 now. The second person offers you 25 annual payments of €15 000 each.
Assuming you can get an annual rate of 5% on your money,which offer should you accept?

Offer 1

Present Value € 200 000

Offer 2

Present Value = ? This is similar to working out Value of mortgage. (P)

$\therefore$ € Payment $= \dfrac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(*formulae and tables*, p.31)

$15000 = P\dfrac{(0.05)(1.05)^{25}}{(1.05)^{25} - 1}$

$15000 = (0.07095)\,P$

$P = \dfrac{15\,000}{0.07095} = €\,211\,409$

The 2nd Offer is best

7. Malcolm needs €400 per month, for 3 years, while he studies at college. What amount of money do his parents need to invest, at 6.6% p.a. compounded monthly, to provide the money that Malcolm needs?

i = AER
r = MER

$r = \sqrt[12]{1+i} - 1$

3 years = 36 months

$P = \frac{F}{(1+i)^t}$

$S_n = \frac{a(1-R^n)}{1-R}$

What is Present Value of €400 pm. for 3 years at 6.6% p.a.?

$i = AER = 6.6\% \Rightarrow MER = r = \sqrt[12]{1.066} - 1$
$= 0.53\%$

$P = 400 + \frac{400}{1.0053} + \frac{400}{1.0053^2} + \ldots + \frac{400}{1.0053^{17}}$

$a = 400$, $n = 18$, Ratio, $R = 1/1.0053$

$P = S_{18} = \frac{400\left(1 - \left(\frac{1}{1.0053}\right)^{18}\right)}{\left(1 - \frac{1}{1.0053}\right)} = €\,6886.25$

7. Malcolm needs €400 per month, for 3 years, while he studies at college. What amount of money do his parents need to invest, at 6.6% p.a. compounded monthly, to provide the money that Malcolm needs?

MER?

$r = \sqrt[12]{1+i} - 1$

3 years = 36 months

$\therefore$ € Payment $= \frac{€\text{ Mortgage }(i)(1+i)^n}{(1+i)^n - 1}$

(*formulae and tables*, p.31)

What is Present Value of €400 pm. for 3 years at 6.6% p.a.? This is similar to working out the Value of a mortgage (P).

$i = AER = 6.6\% \Rightarrow MER = r = \sqrt[12]{1.066} - 1$
$= 0.53\%$

$400 = \frac{P(0.0053)(1.0053)^{36}}{(1.0053)^{36} - 1}$

$400 = (0.0306)P$

$P = \frac{400}{0.0306} = €\,13078.17$