Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Cycle Final Examination
Sample Paper

Mathematics

Higher Level

2 hours

270 marks

Examination Number

Day and Month of Birth

For example, 3rd February
is entered as 0302

Centre Stamp

Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write your answers into this booklet. There is space for extra work at the back of the booklet. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include the appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1   **(Suggested maximum time: 5 minutes)**

(a)   **(i)**   Write the numbers **3**, **9**, and **25** into the three empty boxes below to make the mathematical statement true. Use each number only once.

$$\frac{\boxed{}}{5} + \frac{\boxed{}}{\boxed{}} = \frac{24}{25}$$

        (ii)   Write the numbers **3**, **5**, **9**, and **25** into the empty boxes below so that the **difference** between the two fractions is as large as possible. Use each number only once.

$$\frac{\boxed{}}{\boxed{}} - \frac{\boxed{}}{\boxed{}}$$

(b)   A positive whole number has exactly 4 factors. One of the factors is 9.
Work out the number.

Question 2   (Suggested maximum time: 10 minutes)

Millie bakes cakes and sells them at the local market.

(a) Millie needs 4 eggs to make each cake. She has 28 eggs.
How many cakes can she make?

(b) Millie makes a filling for her cakes using only butter and sugar.
The ratio of the weight of butter to sugar is $5 : 7$.

One day, Millie makes a total of 2·4 kg of filling.
Work out how many **grams** of **sugar** Millie used to make this filling.

(c) Millie is buying flour at her local shop. The shop has two special offers:

Special Offer A	Special Offer B
1 kg bags: €3·50 each	1·5 kg bags: €5 each
Special offer: 3 bags for the price of 2	Special offer: 20% off

Millie wants to buy 6 kg of flour. Work out which offer, **A** or **B**, will give her the better value.

Better value: **Offer A**   **Offer B**
(tick (✓) **one** box only)   ☐   ☐
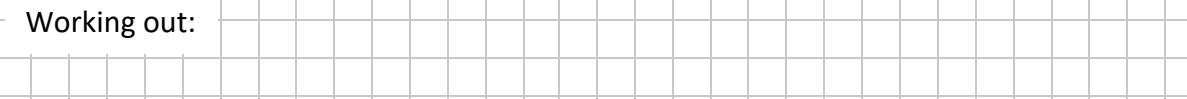
Working out:

(d) Millie sells each cake for €7·50.
This gives her a profit of 20%.

Work out how much it **costs** Millie to make each cake.
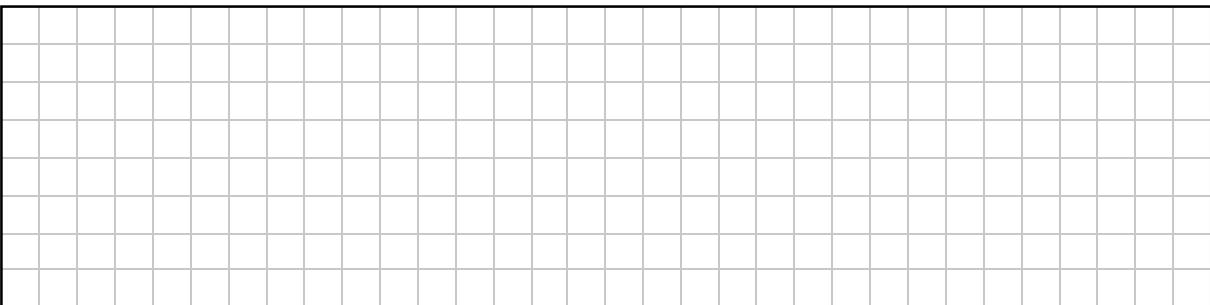
(e) Millie has €3000 in a special savings account.
It has an interest rate of 2·5% per year for 4 years, compounded annually.
She does not put any money in or take any money out of the account over the 4 years.

Work out the **total** amount in the account after the 4 years.
Give your answer correct to the nearest cent.
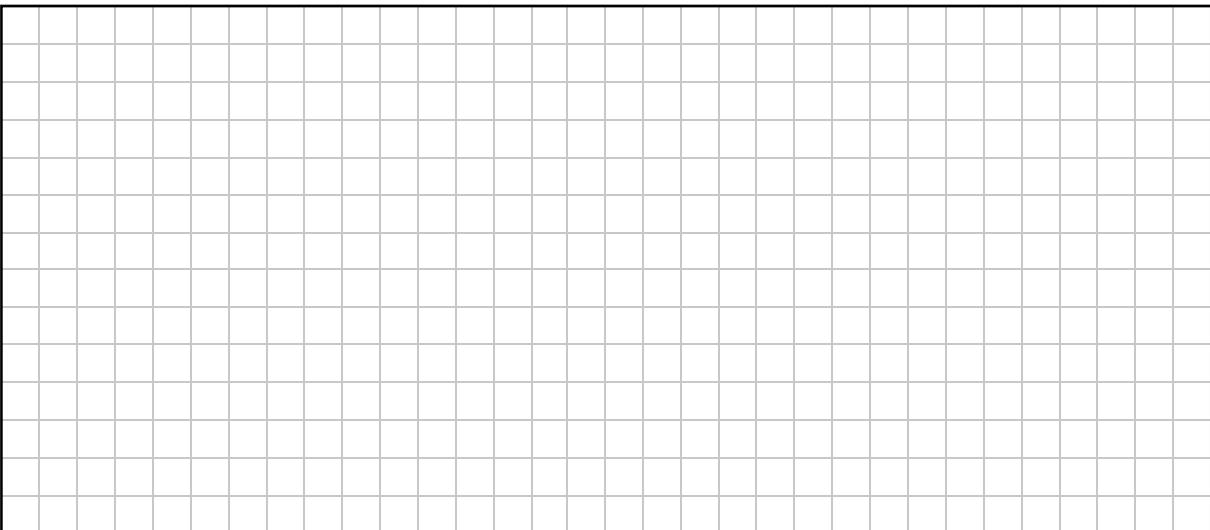
Question 3   **(Suggested maximum time: 10 minutes)**

The diagram below shows the percentage of their total sleep that men and women spend in deep sleep, depending on their age in years, A.


Age, in years (A)
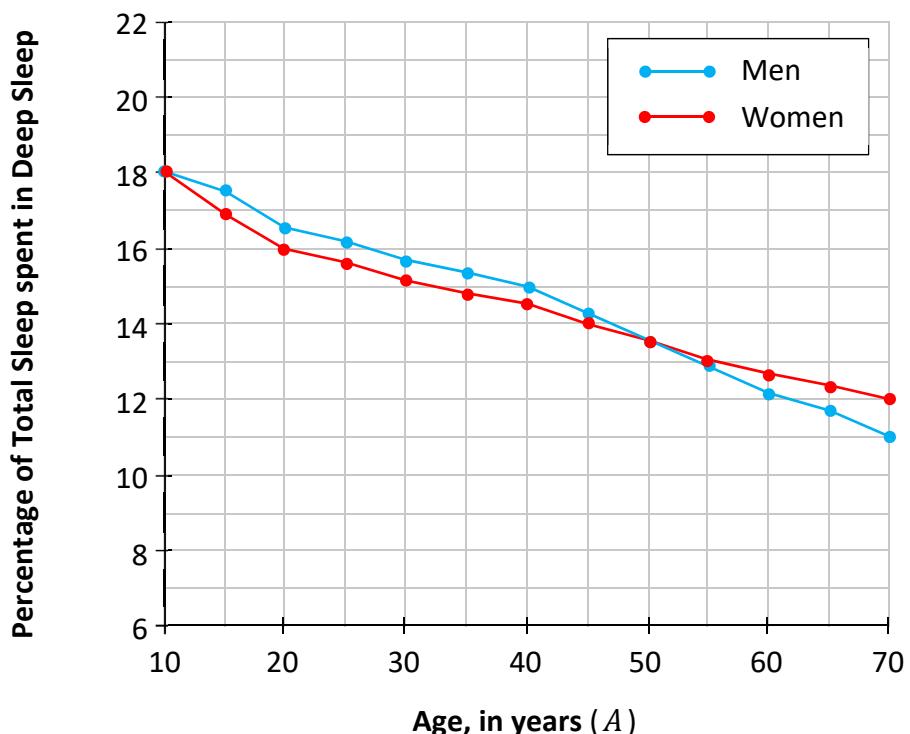
Gina is a 20-year-old woman.

(a) Use the diagram to estimate the percentage of her total sleep that Gina spends in deep sleep. Show your work on the graph.

(b) Gina sleeps an average of 8 hours in total each night.
Work out how many hours Gina spends in deep sleep on average **each week** (7 nights).

(c) Use the diagram to fill in the inequality in A below to show the age range for which **women** spend a **lower** percentage of their sleep in deep sleep than men do.

$$\boxed{} < A < \boxed{}$$

(d) The data in the survey was collected from 6 billion nights of sleep, where a billion is a thousand million. Write 6 billion in the form $a \times 10^n$, where $1 \le a < 10$ and $n \in \mathbb{N}$.

Phillippe uses a **linear** model to estimate the percentage of their total sleep that men spend in deep sleep, from 42 to 66 years of age. Some of his results are in the table below.

(e) Complete the table below so that the percentages follow a linear pattern.
Show your working out.

Age, A (years)	40	45	50	55	60
Percentage of Total Sleep spent in Deep Sleep	15·0		13·6		12·2

(f) Using the values in the table above, Phillippe writes the percentage of sleep spent in deep sleep as a function of age, A, for $40 \le A \le 60$. The graph of this function is a line with a slope of -0.14.

Explain what -0.14 means in the context of this question.

Question 4   **(Suggested maximum time: 15 minutes)**

The co-ordinate diagram below shows the triangle ABC.
The point A has co-ordinates $(4, 2)$.
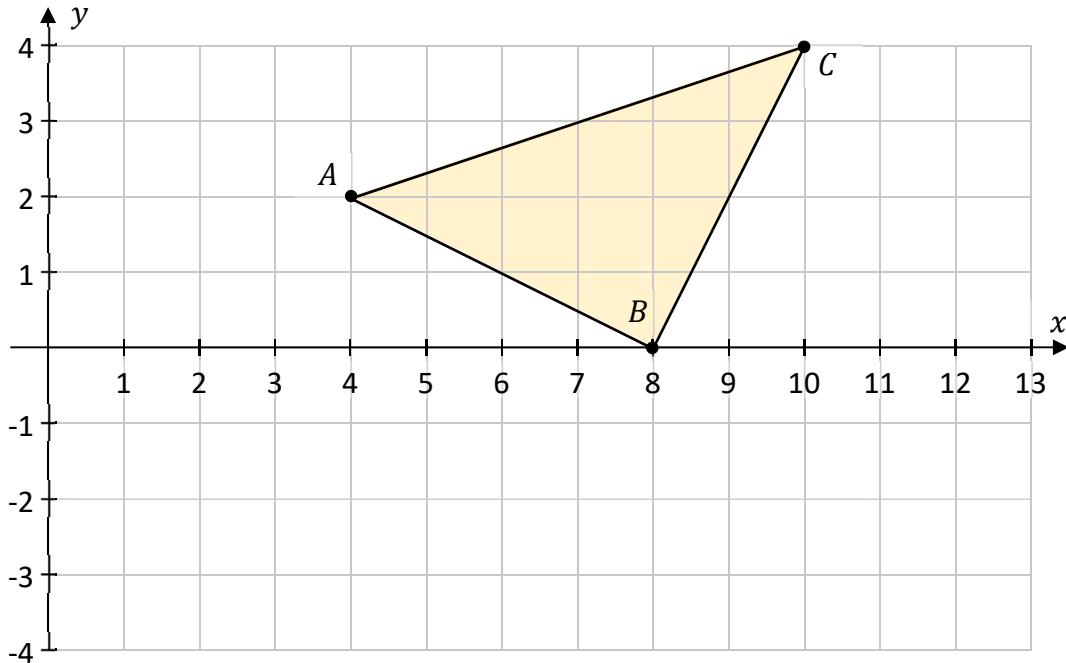
(a) Write down the co-ordinates of the point B and the point C.

$$B = (\qquad , \qquad) \qquad\qquad C = (\qquad , \qquad)$$

(b) The table below shows the equations of the lines AC and AB, where $m, k \in \mathbb{Q}$.
Work out the value of m and the value of k.

Line AC		Line AB	
Equation:	$y = mx + \dfrac{2}{3}$	Equation:	$y = -\dfrac{1}{2}x + k$
Answer:	$m =$ _____	Answer:	$k =$ _____

(c) Show that the **area** of the triangle ABC is 10 square units.
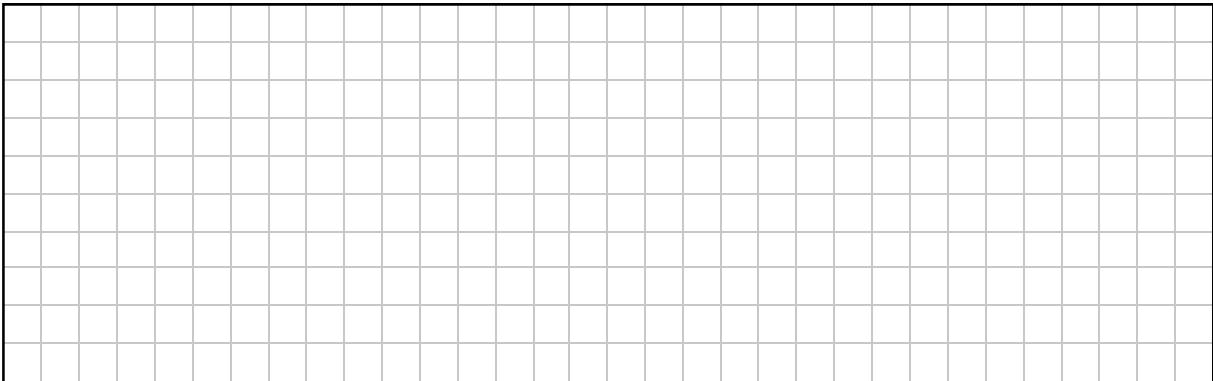
The triangle $A'B'C'$ is the image of ABC under **axial symmetry** in the x axis.

**(d)** **Draw** the triangle $A'B'C'$ on the co-ordinate diagram.
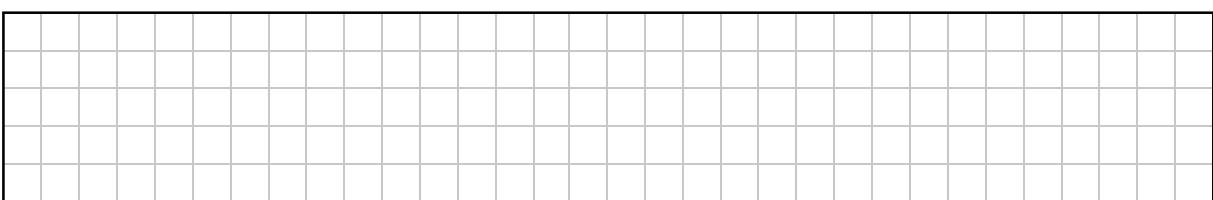
(e) Which of the points A, B, or C is in the **intersection** of the two triangles ABC and $A'B'C'$?

Answer = [   ]

(f) The point (p, s) lies **inside** the triangle ABC, where $p, s \in \mathbb{R}$.

Use this fact to write down the co-ordinates of a point that **must** lie **inside** the triangle $A'B'C'$, giving your answer in terms of p and s.

Answer = (,   )
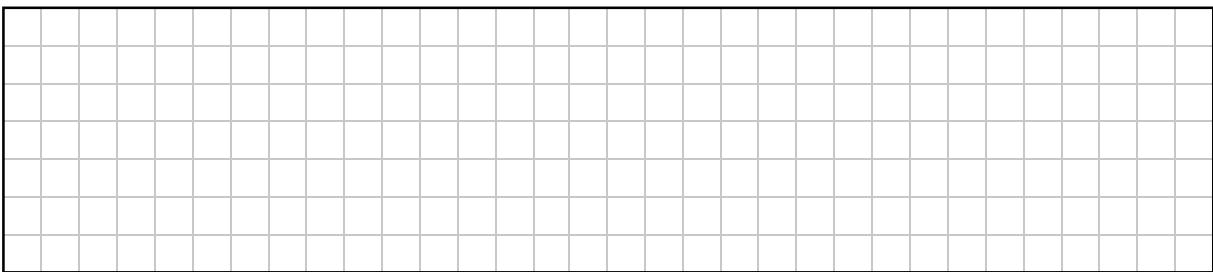
Question 5 (Suggested maximum time: 5 minutes)

In the co-ordinate diagram below, 16 points are marked with a dot (●).
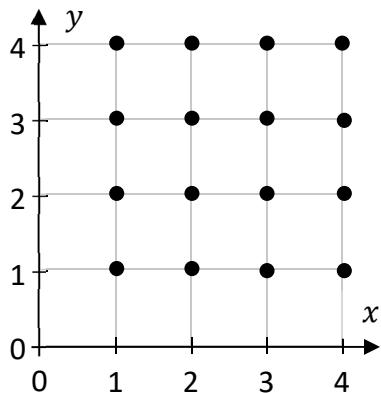
(a) Louise picks 1 point at random from the 16 points marked with a dot in the diagram.
She then finds the equation of the line that goes through this point and through $(0, 0)$.

Find the **probability** that Louise's line has a slope that is **greater than 1**.

(b) How many of the 16 points marked with a dot in the diagram are a distance of **exactly 5 units**
from the point $(0, 0)$?

Question 6   **(Suggested maximum time: 10 minutes)**

(a)   The diagram below shows a quadrilateral $WXYZ$, as well as its diagonals $[WY]$ and $[XZ]$.

      Without measuring, perform **constructions** on the diagram below to show that the diagonals **bisect** each other. Show any necessary construction lines clearly.
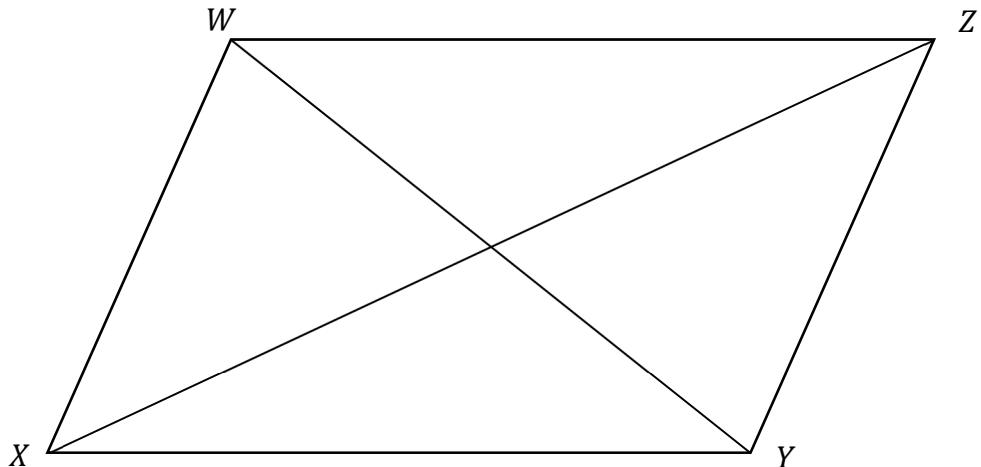
This question continues on the next page.

(b) The diagram below shows the parallelogram $ABCD$. Its diagonals meet at the point O. In this parallelogram, $|\angle DAB| = |\angle ABC|$.

Prove that the triangle ABO is **isosceles**.
Give a reason for each statement that you make in your proof.

Hint: it may help to show that the triangles DAB and CBA are congruent.
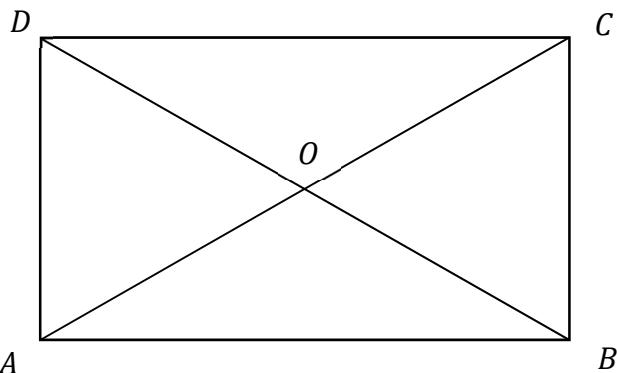
Question 7 (Suggested maximum time: 5 minutes)

(a) A, B, and C are sets.
Complete the tables below by **shading in** each of the given sets in the Venn diagram.

Set	$A \cup B$
Venn diagram	
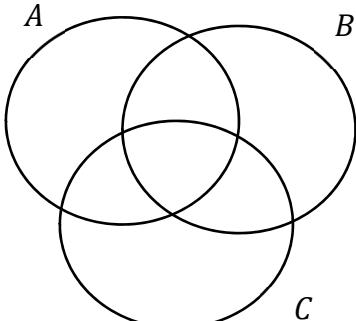
Set	$A \backslash (B \cap C)$
Venn diagram	
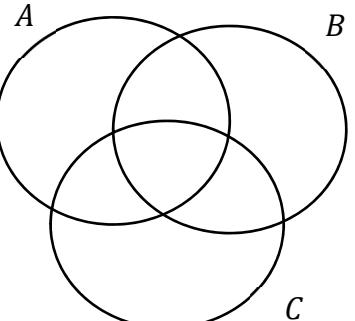
(b) P and Q are two other sets. P is a **subset** of Q.

(i) Write an **X** in the region of the Venn diagram below which must contain **no** elements.
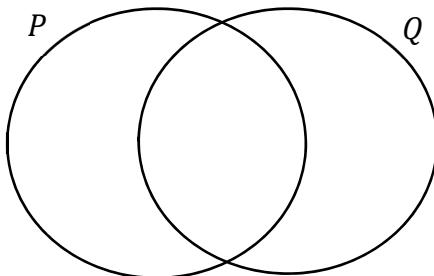
(ii) Put a tick (✓) in the correct box to show which statement **must** be true.
Tick **one** box only. Explain your answer.

$\# P \leq \# Q$ $\# P = \# Q$ $\# P \geq \# Q$

☐   ☐   ☐

Explanation:

Question 8

An **equilateral** triangle XYZ has sides of length 6 cm.

(a) Write down the size of the angle $\angle XYZ$.

(b) Work out the length of the **perimeter** of the triangle XYZ.

(c) Maria plays a game using the triangle XZY. She flips a coin and moves the counter in the direction XYZ, as shown in the diagram below, using the following rule:

- if she gets **heads**, she moves the counter 6 cm
- if she gets **tails**, she moves the counter 12 cm.
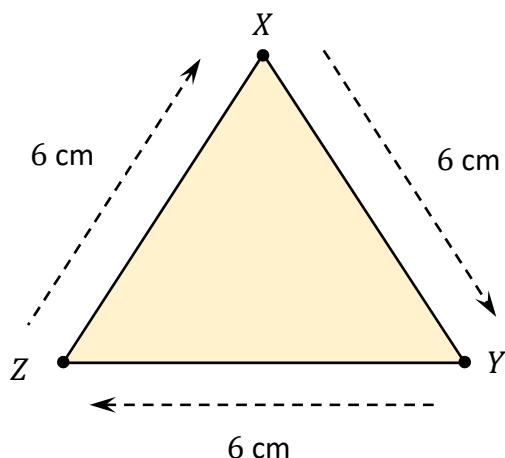
At the start of the game, the counter is at the point X.

Maria gets **tails** on each of the first **4 flips** of the coin.
Which point is the counter at after the first 4 flips of the coin?

(d) Jodie makes a solid shape using equilateral triangles as faces.
It has E edges and F faces, where E, $F \in \mathbb{N}$. For Jodie's shape:

$$\frac{8F}{5} - E = 2$$

$$3F = 2E$$

Solve these simultaneous equations to find the value of E and the value of F.

Question 9   **(Suggested maximum time: 15 minutes)**

(a) A can in the shape of a cylinder has a radius of $3{\cdot}6$ cm and a height of 10 cm.
Work out the **volume** of the can. Give your answer in cm^3, correct to two decimal places.

24 of these cans are to be packed into a closed rectangular box.

(b) The cans will be arranged inside the box as follows:
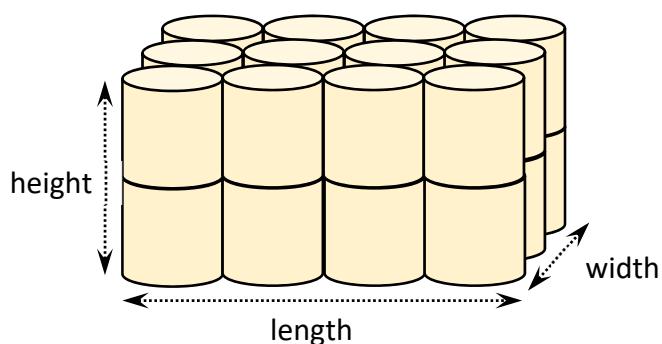
(i) Write down the height, the length, and the width of the smallest rectangular box that will be needed for these 24 cans. One is already done for you.

height = [ ]   length = | 28·8 cm | width = [ ]

(ii) Work out the **volume** of this box.

(iii) Work out the **percentage** of the volume of this box that is taken up by the 24 cans. Give your answer correct to one decimal place.
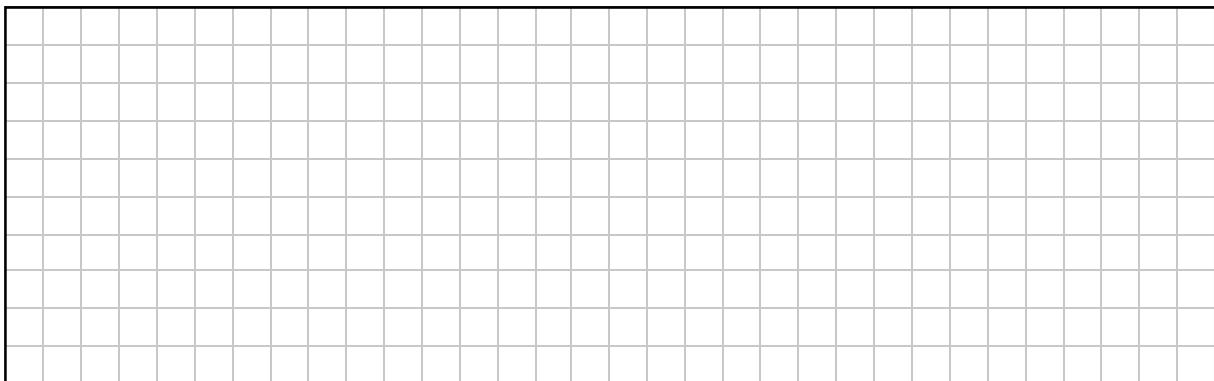
There are a number of different ways of arranging the 24 cans so that they can be packed into a rectangular box. The dimensions of the box may be different for different arrangements.

(c) Find the dimensions of the rectangular box required for a **different** arrangement of the 24 cans. Show your working out.

height = ☐   length = ☐   width = ☐
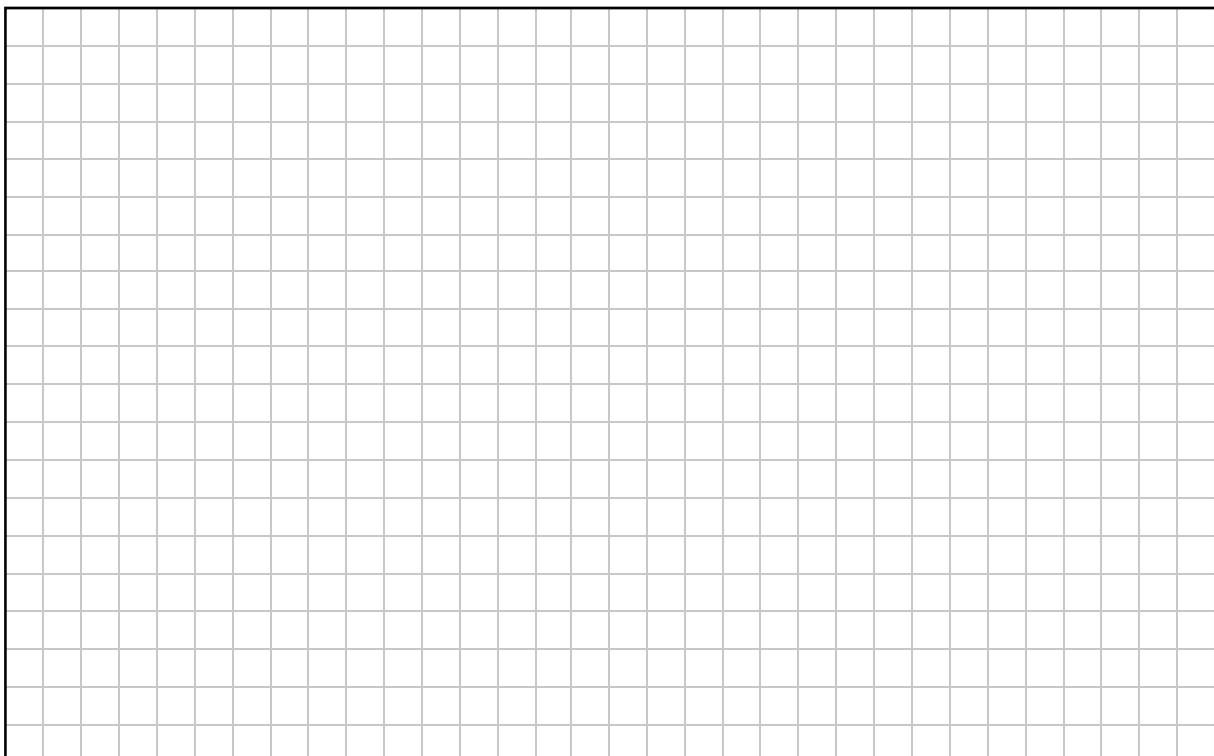
Question 10   **(Suggested maximum time: 5 minutes)**

(a)   Write three integers into the following boxes so that the three numbers have:

- a mode of 2
- a mean of 5

Answer: ☐ , ☐ , and ☐
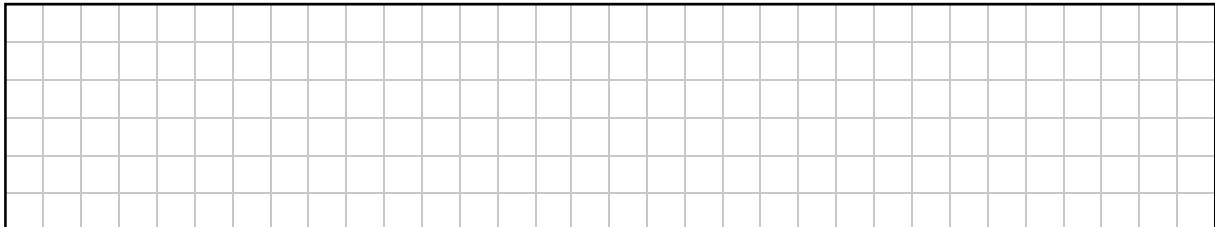
(b)   Write five integers into the following boxes so that the five numbers have:

- a mode of 4
- a median of 4
- a mean of 5
- a range of 12

Answer: ☐ , ☐ , ☐ , ☐ , and ☐
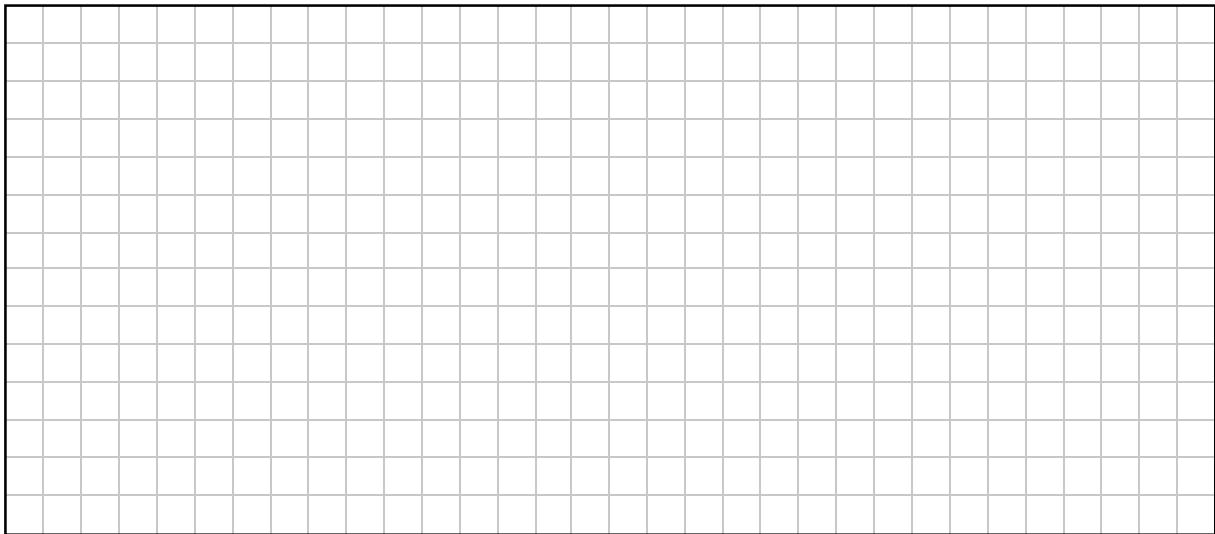
Question 11   **(Suggested maximum time: 5 minutes)**

(a) Find the value of $\dfrac{2p-1}{\sqrt{p^2+15}}$ when $p = -7$.

(b) Factorise fully $5fh - 2h^2 - 6h + 15f$.

(c) Write the following as a single fraction.

$$\frac{5}{2x+1} - \frac{x}{4}$$

Question 12   **(Suggested maximum time: 5 minutes)**

The co-ordinate diagram below shows the graph of the function $y = \cos(x)$, for $0° \leq x \leq 90°$.
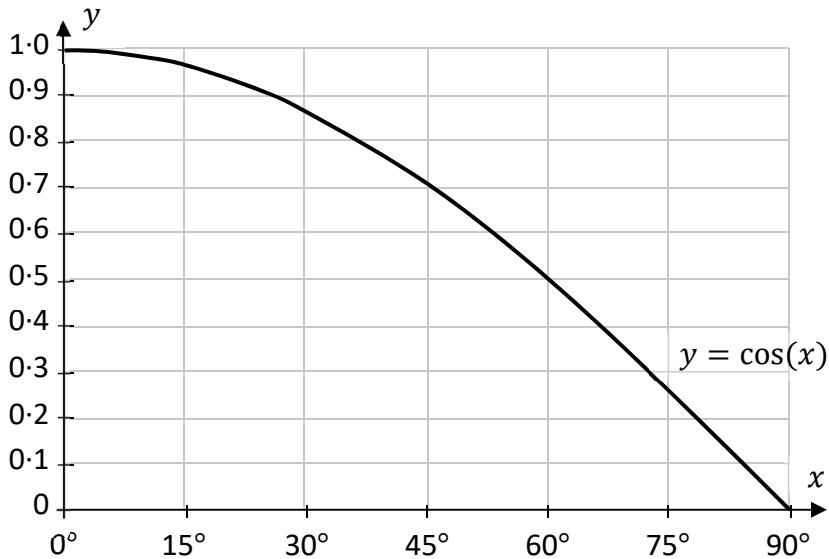
(a) Use a calculator to work out the value of $\sin(60°)$, correct to one decimal place.

(b) **Draw** the graph of $y = \sin(x)$ on the diagram above, using the same axes, scales, and domain. Note that $\sin(0°) = 0$ and $\sin(90°) = 1$.

(c) Using your graph from part **(b)**, estimate the **point of intersection** of $y = \cos(x)$ and $y = \sin(x)$, for $0° \leq x \leq 90°$.

Answer = (,   )

Question 13

The photograph on the right shows a house.
Part of the roof of this house is shown in the
diagram below.

AB is perpendicular to DC.
$|AD| = 11 \cdot 8$ m and $|DB| = 2$ m.
$|\angle CAD| = 20°$ and $|\angle DBC| = x°$.
Note: $\angle ACB$ is **not** a right angle.

Use **trigonometry** to work out the value of x.
Give your answer correct to the nearest whole number.
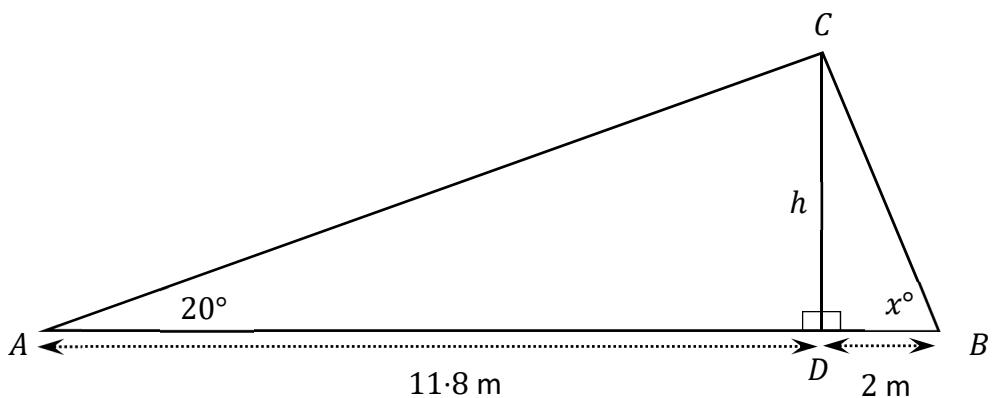
C

h

20°   *x°*

A ←--→ *B*

           11·8 m   *D*   2 m

Source of image: www.interiordesign.net. Altered.

Question 14   **(Suggested maximum time: 5 minutes)**

In the diagram below, the length of each of the sides is given in terms of x, where $x \in \mathbb{N}$.

$x + 4$

$x - 5$

$x - 4$
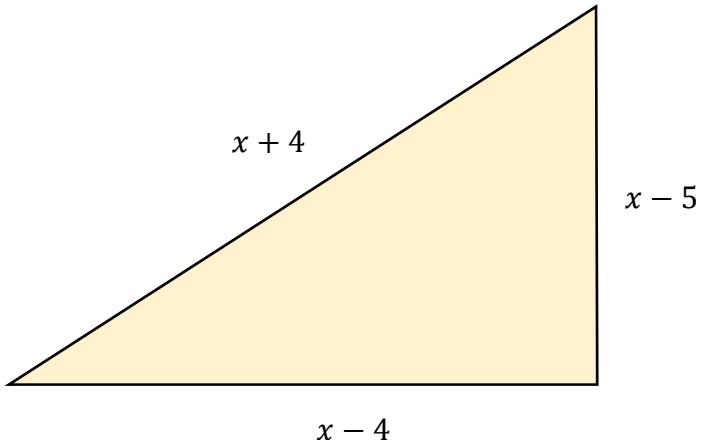
Show that there is only **one** value of x for which this triangle is right-angled.

Page for extra work.
Label any extra work clearly with the question number and part.

Do not write on this page

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Mathematics

2 hours